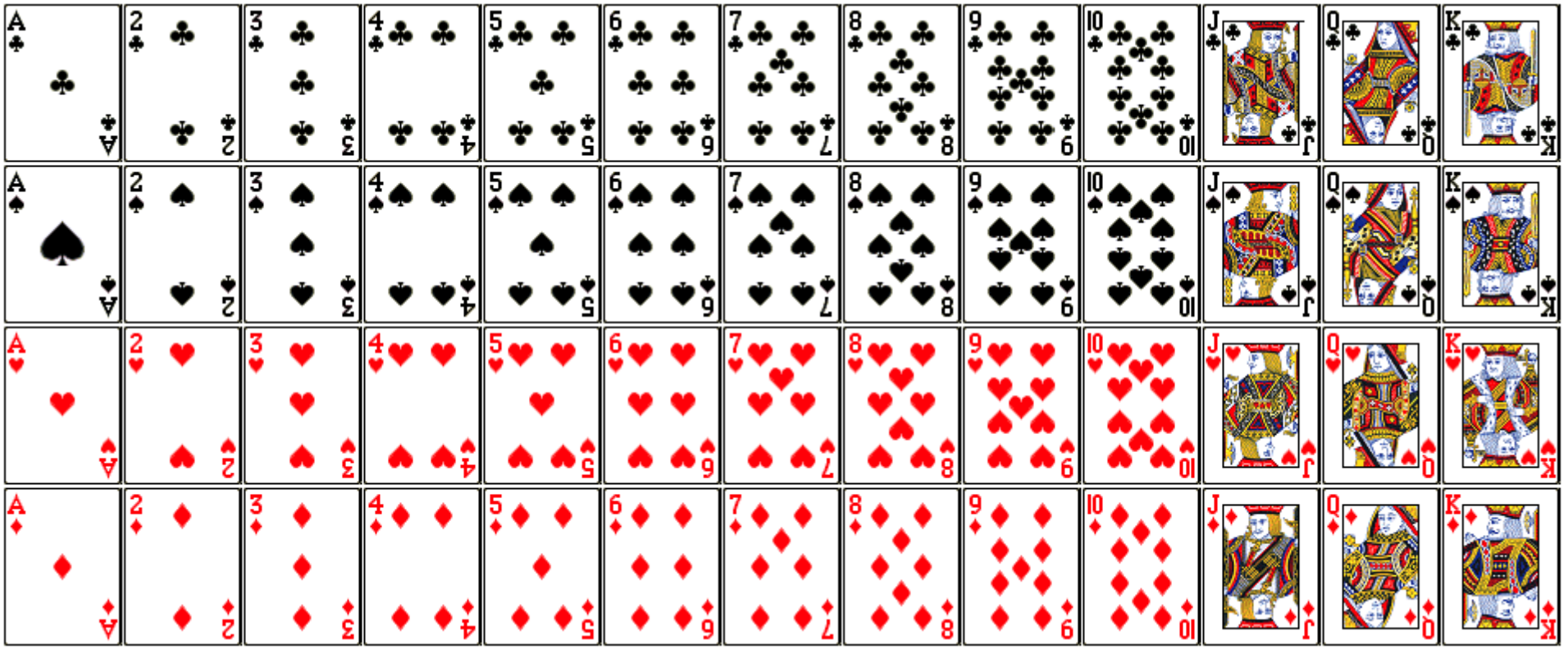
12.1 Introduction to Counting Methods



In this section, we will count things.

We will start with single collections, then will work with compound collections.

How many sides does a coin have?

How many sides does a die have (plural: dice)?

How many cards are in a deck?

A coin has 2 sides, Heads and Tails.

A die (plural: dice) has 6 sides: 1,2,3,4,5,6

There are 52 cards in a deck of cards.
- 4 suits: hearts, diamonds, clubs, spades
- 13 face cards: Ace, 2, 3, … 10,
  Jack, Queen, King

Six people { A, B, C, D, E, F } enter a raffle for which two will win the same prize.

What are the ways two people can be chosen?

How many different ways are there for two winners to be chosen?

Six people { A, B, C, D, E, F } enter a raffle for which two will win the same prize.

What are the ways two people can be chosen?

AB AC AD AE AF BC BD BE BF
CD CE CF DE DF EF

How many different ways are there for two winners to be chosen?

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Flip a coin {H,T} once and then roll a die {1, …, 6}. How many different outcomes are there?

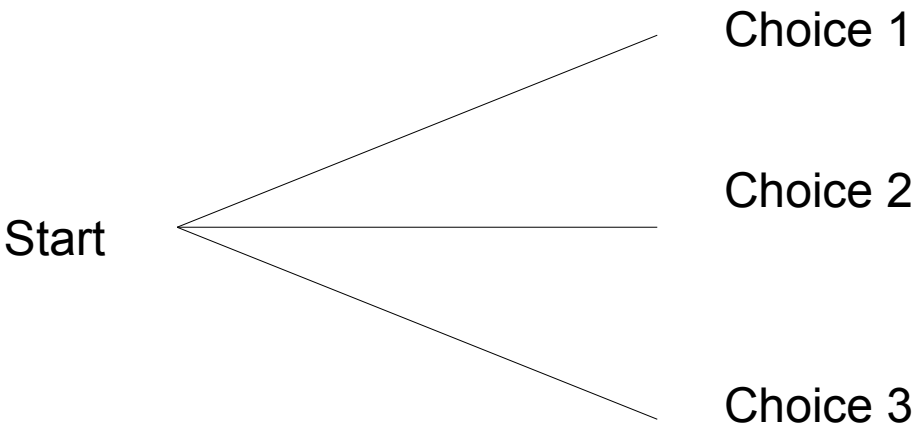
Flip a coin {H,T} once and then roll a die {1, …, 6}. How many different outcomes are there?

H1 H2 H3 H4 H5 H6
T1 T2 T3 T4 T5 T6

12 different outcomes

There is a graphical way to organize and count.

A **tree diagram** is a visual method for each new choice at a step we get a new branch. Work from left to right.



Example: Flip one coin and then roll a die

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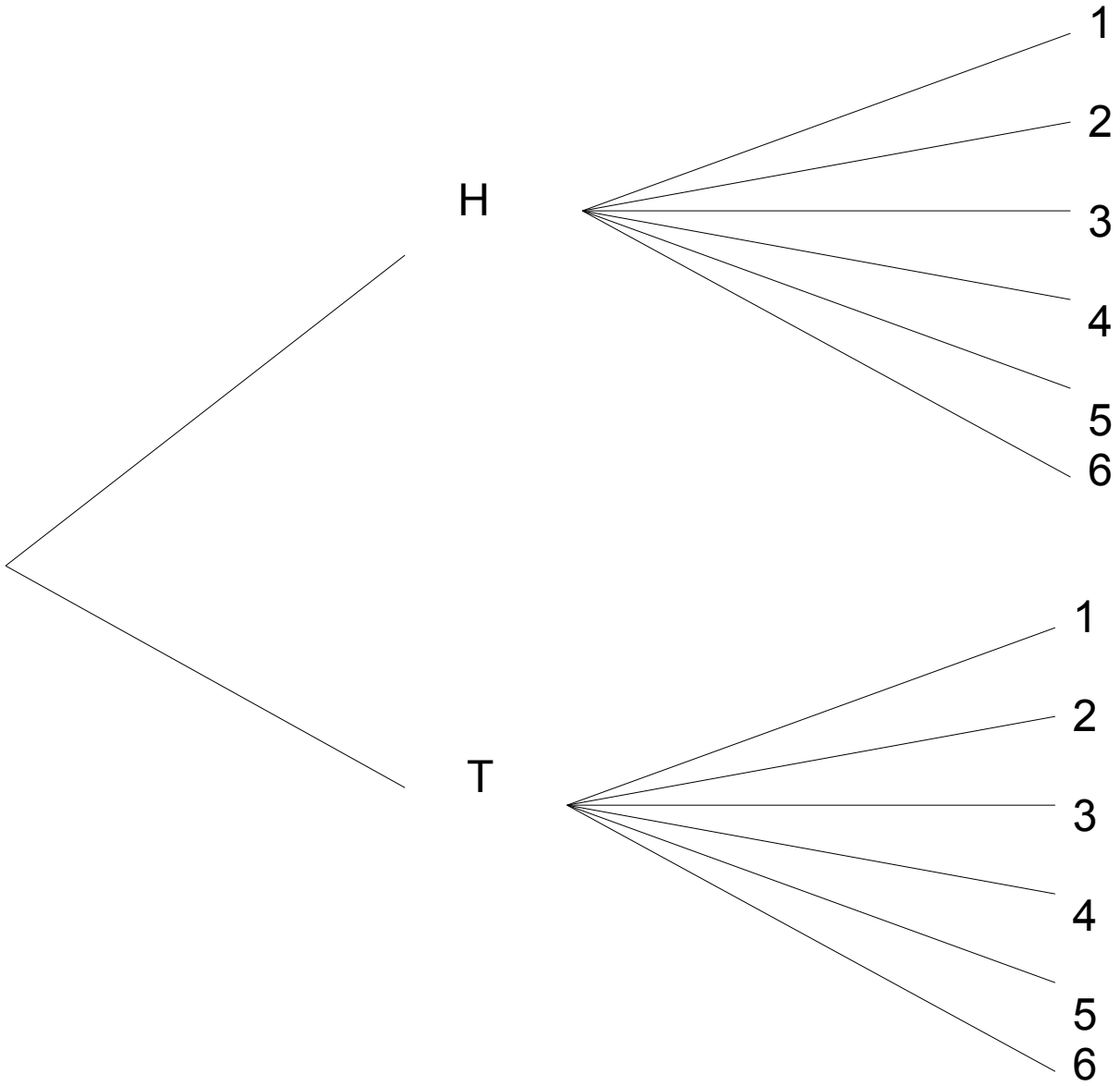
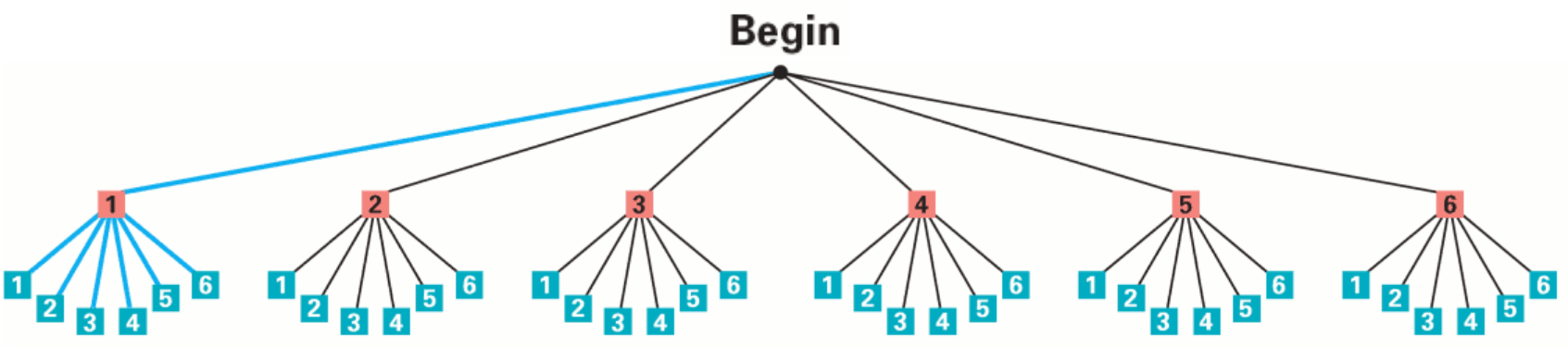


How many ways are there to roll 2 dice?



When we rolled the dice, the same number could come up twice.

When we picked winners from a raffle, the same person could not be picked twice.

This leads to two ideas.

In a selection where repetition is allowed, the phrase **with repetition** is used.

In a selection where repetition is not allowed, the phrase **without repetition** is used.

Examples:

1) Rolling a die.

2) Eating two pieces of candy, each of a different color, one at a time.

3) Pulling a marble from a bag and returning it.

4) Pulling a marble from a bag and leaving it out.

Draw the tree diagram for pulling 3 marbles from a bag { R, G, B }, without repetition.

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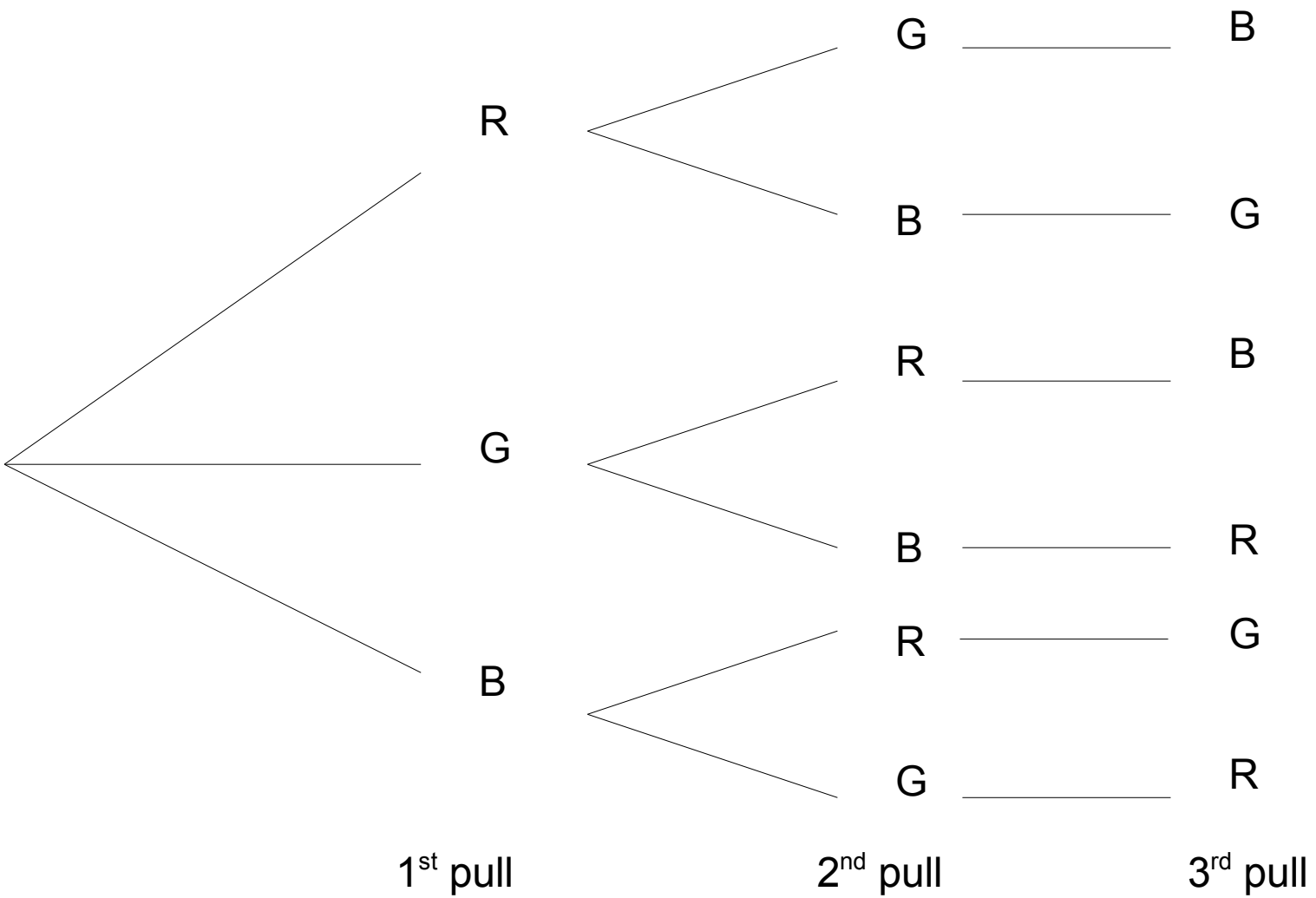
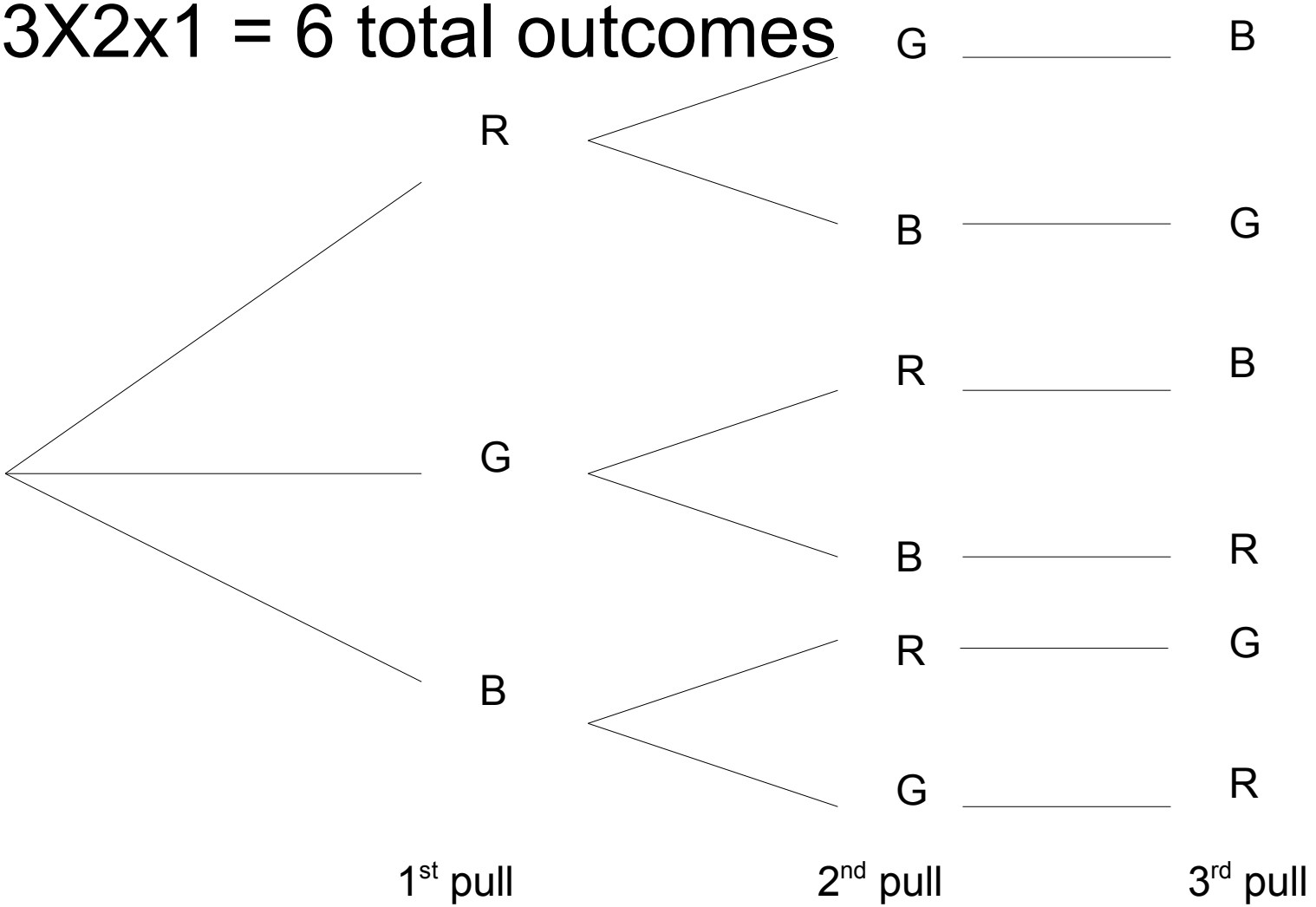


The **Multiplication Principle**:

If an experiment has N choices for the 1^{st} layer of choices and M for the 2^{nd} layer of choices,
then there are NxM total choices.

With more layers, multiply all of the number of choices at each layer together.

3 for 1st layer.
2 for 2nd layer.
1 for 3rd layer
3X2x1 = 6 total outcomes



Example:

For a car, you have 9 choices of color Black, White, Brown, and ROYGBV and two types of interior {Leather, Cloth} to choose from.

How many branches for the choice of color?

How many branches for the choice of interior?

How many different combinations?

Example: An eyewitness saw a getaway car with the license plate “OH ####” where the numbers are {5, 7, 8, 0} in some order.

Find how many different license plates satisfy this by counting the branches on a tree diagram.